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## COMMON NEIGHBORHOOD AND NEAR COMMON NEIGHBORHOOD $n$ -SIGRAPHS

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### Abstract

In this paper we introduced the new notions eccentric and super eccentric symmetric  $n$ -sigraph of a symmetric  $n$ -sigraph and its properties are obtained. Also, we obtained the structural characterizations of these notions. Further, we presented some switching equivalent characterizations.

### 1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [5]. We consider only finite, simple graphs free from self-loops.

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Let  $n \geq 1$  be an integer. An  $n$ -tuple  $(a_1, a_2, \dots, a_n)$  is *symmetric*, if  $a_k = a_{n-k+1}$ ,  $1 \leq k \leq n$ . Let  $H_n = \{(a_1, a_2, \dots, a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$  be the set of all symmetric  $n$ -tuples. Note that  $H_n$  is a group under coordinate wise multiplication, and the order of  $H_n$  is  $2^m$ , where  $m = \lceil \frac{n}{2} \rceil$ .

A *symmetric  $n$ -sigraph* (*symmetric  $n$ -marked graph*) is an ordered pair  $S_n = (G, \sigma)$  ( $S_n = (G, \mu)$ ), where  $G = (V, E)$  is a graph called the *underlying graph* of  $S_n$  and  $\sigma : E \rightarrow H_n$  ( $\mu : V \rightarrow H_n$ ) is a function.

In this paper by an  *$n$ -tuple/ $n$ -sigraph/ $n$ -marked graph* we always mean a symmetric  $n$ -tuple/*symmetric  $n$ -sigraph*/*symmetric  $n$ -marked graph*.

An  $n$ -tuple  $(a_1, a_2, \dots, a_n)$  is the *identity  $n$ -tuple*, if  $a_k = +$ , for  $1 \leq k \leq n$ , otherwise it is a *non-identity  $n$ -tuple*. In an  $n$ -sigraph  $S_n = (G, \sigma)$  an edge labelled with the identity  $n$ -tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an  $n$ -sigraph  $S_n = (G, \sigma)$ , for any  $A \subseteq E(G)$  the  *$n$ -tuple  $\sigma(A)$*  is the product of the  $n$ -tuples on the edges of  $A$ .

In [11], the authors defined two notions of balance in  $n$ -sigraph  $S_n = (G, \sigma)$  as follows (See also R. Rangarajan and P.S.K.Reddy [7]).

**Definition.** Let  $S_n = (G, \sigma)$  be an  $n$ -sigraph. Then,

- (i)  $S_n$  is *identity balanced* (or  *$i$ -balanced*), if product of  $n$ -tuples on each cycle of  $S_n$  is the identity  $n$ -tuple, and
- (ii)  $S_n$  is *balanced*, if every cycle in  $S_n$  contains an even number of non-identity edges.

**Note:** An  $i$ -balanced  $n$ -sigraph need not be balanced and conversely.

The following characterization of  $i$ -balanced  $n$ -sigraphs is obtained in [11].

**Theorem 1.1 :** (*E. Sampathkumar et al. [11]*). An  $n$ -sigraph  $S_n = (G, \sigma)$  is  $i$ -balanced if, and only if, it is possible to assign  $n$ -tuples to its vertices such that the  $n$ -tuple of each edge  $uv$  is equal to the product of the  $n$ -tuples of  $u$  and  $v$ .

In [11], the authors also have defined switching and cycle isomorphism of an  $n$ -sigraph  $S_n = (G, \sigma)$  as follows: (See also [6], [8-10], [13-23]).

Let  $S_n = (G, \sigma)$  and  $S'_n = (G', \sigma')$ , be two  $n$ -sigraphs. Then  $S_n$  and  $S'_n$  are said to be *isomorphic*, if there exists an isomorphism  $\phi : G \rightarrow G'$  such that if  $uv$  is an edge in  $S_n$  with label  $(a_1, a_2, \dots, a_n)$  then  $\phi(u)\phi(v)$  is an edge in  $S'_n$  with label  $(a_1, a_2, \dots, a_n)$ .

Given an  $n$ -marking  $\mu$  of an  $n$ -sigraph  $S_n = (G, \sigma)$ , *switching*  $S_n$  with respect to  $\mu$  is the operation of changing the  $n$ -tuple of every edge  $uv$  of  $S_n$  by  $\mu(u)\sigma(uv)\mu(v)$ . The  $n$ -sigraph obtained in this way is denoted by  $\mathcal{S}_\mu(S_n)$  and is called the  $\mu$ -switched  $n$ -sigraph or just *switched  $n$ -sigraph*.

Further, an  $n$ -sigraph  $S_n$  *switches* to  $n$ -sigraph  $S'_n$  (or that they are *switching equivalent* to each other), written as  $S_n \sim S'_n$ , whenever there exists an  $n$ -marking of  $S_n$  such that  $\mathcal{S}_\mu(S_n) \cong S'_n$ .

Two  $n$ -sigraphs  $S_n = (G, \sigma)$  and  $S'_n = (G', \sigma')$  are said to be *cycle isomorphic*, if there exists an isomorphism  $\phi : G \rightarrow G'$  such that the  $n$ -tuple  $\sigma(C)$  of every cycle  $C$  in  $S_n$  equals to the  $n$ -tuple  $\sigma(\phi(C))$  in  $S'_n$ . We make use of the following known result (see [11]).

**Theorem 1.2 :** emph(**E. Sampathkumar et al.** [11]). Given a graph  $G$ , any two  $n$ -sigraphs with  $G$  as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

Let  $S_n = (G, \sigma)$  be an  $n$ -sigraph. Consider the  $n$ -marking  $\mu$  on vertices of  $S$  defined as follows: each vertex  $v \in V$ ,  $\mu(v)$  is the product of the  $n$ -tuples on the edges incident at  $v$ . *Complement* of  $S$  is an  $n$ -sigraph  $\overline{S}_n = (\overline{G}, \sigma')$ , where for any edge  $e = uv \in \overline{G}$ ,  $\sigma'(uv) = \mu(u)\mu(v)$ . Clearly,  $\overline{S}_n$  as defined here is an  $i$ -balanced  $n$ -sigraph due to Theorem 1.1.

## 2. Common Neighborhood $n$ -Sigraph of an $n$ -Sigraph

The common neighborhood graph  $\mathcal{CN}(G)$  of  $G = (V, E)$  is a graph with  $V(\mathcal{CN}(G)) = V(G)$  and any two vertices  $u$  and  $v$  in  $\mathcal{CN}(G)$  are joined by an edge if and only if the vertices  $u$  and  $v$  in  $G$  have at least one common neighbor in the graph  $G$ . This concept were introduced by A. Alwardi et al. [1].

Motivated by the existing definition of complement of an  $n$ -sigraph, we extend the notion of common neighborhood to  $n$ -sigraphs as follows:

The *common neighborhood  $n$ -sigraph*  $\mathcal{CN}(S_n)$  of an  $n$ -sigraph  $S_n = (G, \sigma)$  is an  $n$ -sigraph whose underlying graph is  $\mathcal{CN}(G)$  and the  $n$ -tuple of any edge  $uv$  is  $\mathcal{CN}(S_n)$  is  $\mu(u)\mu(v)$ , where  $\mu$  is the canonical  $n$ -marking of  $S_n$ . Further, an  $n$ -sigraph  $S_n = (G, \sigma)$  is called common neighborhood  $n$ -sigraph, if  $S_n \cong \mathcal{CN}(S'_n)$  for some  $n$ -sigraph  $S'_n$ . The following result restricts the class of common neighborhood graphs.

**Theorem 2.1 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ , its common neighborhood  $n$ -sigraph  $\mathcal{CN}(S_n)$  is  $i$ -balanced.

**Proof :** Since the  $n$ -tuple of any edge  $uv$  in  $\mathcal{CN}(S_n)$  is  $\mu(u)\mu(v)$ , where  $\mu$  is the canonical  $n$ -marking of  $S_n$ , by Theorem 1.1,  $\mathcal{CN}(S_n)$  is  $i$ -balanced.  $\square$

For any positive integer  $k$ , the  $k^{\text{th}}$  iterated common neighborhood  $n$ -sigraph  $\mathcal{CN}(S_n)$  of  $S_n$  is defined as follows:

$$(\mathcal{CN})^0(S_n) = S_n, (\mathcal{CN})^k(S_n) = \mathcal{CN}((\mathcal{CN})^{k-1}(S_n)).$$

**Corollary 2.2 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$  and any positive integer  $k$ ,  $(\mathcal{CN})^k(S_n)$  is  $i$ -balanced.

The following result characterize  $n$ -sigraphs which are common neighborhood  $n$ -sigraphs.

**Theorem 2.3 :** An  $n$ -sigraph  $S_n = (G, \sigma)$  is a common neighborhood  $n$ -sigraph if, and only if,  $S_n$  is  $i$ -balanced  $n$ -sigraph and its underlying graph  $G$  is a common neighborhood graph.

**Proof :** Suppose that  $S_n$  is  $i$ -balanced and  $G$  is a  $\mathcal{CN}(G)$ . Then there exists a graph  $H$  such that  $\mathcal{CN}(H) \cong G$ . Since  $S_n$  is  $i$ -balanced, by Theorem 1.1, there exists an  $n$ -marking  $\mu$  of  $G$  such that each edge  $uv$  in  $S_n$  satisfies  $\sigma(uv) = \mu(u)\mu(v)$ . Now consider the  $n$ -sigraph  $S'_n = (H, \sigma')$ , where for any edge  $e$  in  $H$ ,  $\sigma'(e)$  is the  $n$ -marking of the corresponding vertex in  $G$ . Then clearly,  $\mathcal{CN}(S'_n) \cong S_n$ . Hence  $S_n$  is a common neighborhood  $n$ -sigraph.

Conversely, suppose that  $S_n = (G, \sigma)$  is a common neighborhood  $n$ -sigraph. Then there exists an  $n$ -sigraph  $S'_n = (H, \sigma')$  such that  $\mathcal{CN}(S'_n) \cong S_n$ . Hence  $G$  is the  $\mathcal{CN}(G)$  of  $H$  and by Theorem 2.1,  $S_n$  is  $i$ -balanced.  $\square$

In [2], the authors remarked that  $\mathcal{CN}(G) \cong G$  if and only if  $G$  is  $K_n$  or  $\overline{K_n}$  or  $C_n$ . We now characterize the signed graphs such that the common neighborhood  $n$ -sigraph and its corresponding  $n$ -sigraph are switching equivalent.

**Theorem 2.4 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ , the common neighborhood  $n$ -sigraph  $\mathcal{CN}(S_n)$  and  $S_n$  are cycle isomorphic if and only if the underlying of  $S_n$  is isomorphic to  $K_n$  or  $\overline{K_n}$  or  $C_n$  and  $S_n$  is  $i$ -balanced.

**Proof :** Suppose  $\mathcal{CN}(S_n) \sim S_n$ . This implies,  $\mathcal{CN}(G) \cong G$  and hence  $G$  is isomorphic to  $K_n$  or  $\overline{K_n}$  or  $C_n$ . Then  $\mathcal{CN}(S_n)$  is  $i$ -balanced and hence if  $S_n$  is  $i$ -unbalanced and its common neighborhood  $n$ -sigraph  $\mathcal{CN}(S_n)$  being  $i$ -balanced can not be switching equivalent to  $S_n$  in accordance with Theorem 1.2. Therefore,  $S_n$  must be  $i$ -balanced.

Conversely, suppose that  $S_n$  is an  $i$ -balanced  $n$ -sigraph with the underlying graph  $G$  is isomorphic to  $K_n$  or  $\overline{K_n}$  or  $C_n$ . Then, since  $\mathcal{CN}(S_n)$  is  $i$ -balanced as per Theorem 2.1 and since  $\mathcal{CN}(G) \cong G$ , the result follows from Theorem 1.2 again.  $\square$

In [4], the authors defined the derived graph of a graph as follows: The derived graph  $\mathcal{DR}(G)$  of  $G = (V, E)$  is a graph with  $V(\mathcal{DR}(G)) = V(G)$  and any two vertices  $u$  and  $v$  in  $\mathcal{DR}(G)$  are joined by an edge if and only if  $d(u, v) = 2$  in graph  $G$ .

We now define the derived  $n$ -sigraph of an  $n$ -sigraphs as follows: The *derived  $n$ -sigraph*  $\mathcal{DR}(S_n)$  of an  $n$ -sigraph  $S_n = (G, \sigma)$  is an  $n$ -sigraph whose underlying graph is  $\mathcal{DR}(G)$  and the  $n$ -tuple of any edge  $uv$  in  $\mathcal{DR}(S_n)$  is  $\zeta(u)\zeta(v)$ , where  $\zeta$  is the canonical marking of  $S_n$ . Further, an  $n$ -sigraph  $S_n = (G, \sigma)$  is called a derived  $n$ -sigraph, if  $S_n \cong \mathcal{DR}(S'_n)$  for some  $n$ -sigraph  $S'_n$ . The following result restricts the class of derived graphs.

**Theorem 2.5 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ , its derived  $n$ -sigraph  $\mathcal{D}\nabla(S_n)$  is  $i$ -balanced.

**Proof :** Since the  $n$ -tuple of any edge  $uv$  in  $\mathcal{DR}(S_n)$  is  $\mu(u)\mu(v)$ , where  $\mu$  is the canonical  $n$ -marking of  $S_n$ , by Theorem 1.1,  $\mathcal{DR}(S_n)$  is  $i$ -balanced.  $\square$

For any positive integer  $k$ , the  $k^{\text{th}}$  iterated derived  $n$ -sigraph  $\mathcal{DR}(S_n)$  of  $S_n$  is defined as follows:

$$(\mathcal{DR})^0(S_n) = S_n, (\mathcal{DR})^k(S_n) = \mathcal{DR}((\mathcal{DR})^{k-1}(S_n)).$$

**Corollary 2.6 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$  and any positive integer  $k$ ,  $(\mathcal{DR})^k(S_n)$  is  $i$ -balanced.

The following result characterize  $n$ -sigraphs which are derived  $n$ -sigraphs.

**Theorem 2.7 :** An  $n$ -sigraph  $S_n = (G, \sigma)$  is a derived  $n$ -sigraph if, and only if,  $S_n$  is  $i$ -balanced  $n$ -sigraph and its underlying graph  $G$  is a derived graph.

**Proof :** Suppose that  $S_n$  is  $i$ -balanced and  $G$  is a  $\mathcal{DR}(G)$ . Then there exists a graph  $H$  such that  $\mathcal{DR}(H) \cong G$ . Since  $S_n$  is  $i$ -balanced, by Theorem 1.1, there exists an  $n$ -marking  $\mu$  of  $G$  such that each edge  $uv$  in  $S_n$  satisfies  $\sigma(uv) = \mu(u)\mu(v)$ . Now consider the  $n$ -sigraph  $S'_n = (H, \sigma')$ , where for any edge  $e$  in  $H$ ,  $\sigma'(e)$  is the  $n$ -marking of the corresponding vertex in  $G$ . Then clearly,  $\mathcal{DR}(S'_n) \cong S_n$ . Hence  $S_n$  is a derived  $n$ -sigraph.

Conversely, suppose that  $S_n = (G, \sigma)$  is a derived  $n$ -sigraph. Then there exists an  $n$ -sigraph  $S'_n = (H, \sigma')$  such that  $\mathcal{DR}(S'_n) \cong S_n$ . Hence  $G$  is the  $\mathcal{DR}(G)$  of  $H$  and by Theorem 2.1,  $S_n$  is  $i$ -balanced.  $\square$

We now characterize the  $n$ -sigraphs for which  $\overline{S_n}$  and  $\mathcal{DR}(S_n)$  are cycle isomorphic.

**Theorem 2.8 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ ,  $\mathcal{DR}(S_n)$  and  $\overline{S_n}$  are cycle isomorphic if and only if diameter of  $G$  is 2.

**Proof :** Suppose  $\mathcal{DR}(S_n) \sim \overline{S_n}$ . This implies,  $\mathcal{DR}(G) \cong \overline{G}$ . Then any pair of non-adjacent vertices is at distance two and hence diameter of  $G$  is 2.

Conversely, suppose that  $S_n$  is any  $n$ -sigraph with diameter of  $G$  is 2. Then,  $\mathcal{DR}(G) \cong \overline{G}$ , and any pair of non-adjacent vertices is at distance two, and these vertices are adjacent in  $\mathcal{DR}(G)$ . Since for any  $n$ -sigraph  $S_n$ , both  $\mathcal{DR}(S_n)$  and  $\overline{S_n}$  are  $i$ -balanced, the result follows by Theorem 1.2.  $\square$

In [2], the authors remarked that  $\mathcal{CN}(G) \cong \mathcal{DR}(G)$  if and only if  $G$  is bipartite. We now characterize the  $n$ -sigraphs for which:  $\mathcal{CN}(S_n)$  and  $\mathcal{DR}(S_n)$  are cycle isomorphic.

**Thwoewm 2.9 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ ,  $\mathcal{CN}(S_n)$  and  $\mathcal{DR}(S_n)$  are cycle isomorphic if and only if the underlying graph of  $S_n$  is bipartite.

**Proof :** Suppose  $\mathcal{CN}(S_n) \sim \mathcal{DR}(S_n)$ . This implies,  $\mathcal{CN}(G) \cong \mathcal{DR}(G)$ . Then  $G$  is bipartite.

Conversely, suppose that  $S_n$  is any  $n$ -sigraph whose underlying graph  $G$  is bipartite. Then,  $\mathcal{CN}(G) \cong \mathcal{DR}(G)$ . Since for any  $n$ -sigraph  $S_n$ , both  $\mathcal{CN}(S_n)$  and  $\mathcal{DR}(S_n)$  are  $i$ -balanced, the result follows by Theorem 1.2.  $\square$

### 3. Near Common Neighborhood $n$ -Sigraph of an $n$ -Sigraph

Motivated by the existing definition of complement of an  $n$ -sigraph, we extend the notion of near common neighborhood to  $n$ -sigraphs as follows:

The *near common neighborhood  $n$ -sigraph*  $\mathcal{NCN}(S_n)$  of an  $n$ -sigraph  $S_n = (G, \sigma)$  is an  $n$ -sigraph whose underlying graph is  $\mathcal{NCN}(G)$  and the  $n$ -tuple of any edge  $uv$  is  $\mathcal{NCN}(S_n)$  is  $\mu(u)\mu(v)$ , where  $\mu$  is the canonical  $n$ -marking of  $S_n$ . Further, an  $n$ -sigraph  $S_n = (G, \sigma)$  is called near common neighborhood  $n$ -sigraph, if  $S_n \cong \mathcal{NCN}(S'_n)$  for some  $n$ -sigraph  $S'_n$ . The following result restricts the class of near common neighborhood graphs.

**Theorem 3.1 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ , its near common neighborhood  $n$ -sigraph  $\mathcal{NCN}(S_n)$  is  $i$ -balanced.

**Proof :** Since the  $n$ -tuple of any edge  $uv$  in  $\mathcal{NCN}(S_n)$  is  $\mu(u)\mu(v)$ , where  $\mu$  is the canonical  $n$ -marking of  $S_n$ , by Theorem 1.1,  $\mathcal{NCN}(S_n)$  is  $i$ -balanced.  $\square$

For any positive integer  $k$ , the  $k^{th}$  iterated near common neighborhood  $n$ -sigraph  $\mathcal{N}\mathcal{C}\mathcal{N}(S_n)$  of  $S_n$  is defined as follows:

$$(\mathcal{N}\mathcal{C}\mathcal{N})^0(S_n) = S_n, (\mathcal{N}\mathcal{C}\mathcal{N})^k(S_n) = \mathcal{N}\mathcal{C}\mathcal{N}((\mathcal{N}\mathcal{C}\mathcal{N})^{k-1}(S_n)).$$

**Corollary 3.2 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$  and any positive integer  $k$ ,  $(\mathcal{N}\mathcal{C}\mathcal{N})^k(S_n)$  is  $i$ -balanced.

The following result characterize  $n$ -sigraphs which are near common neighborhood  $n$ -sigraphs.

**Theorem 3.3 :** An  $n$ -sigraph  $S_n = (G, \sigma)$  is a near common neighborhood  $n$ -sigraph if, and only if,  $S_n$  is  $i$ -balanced  $n$ -sigraph and its underlying graph  $G$  is a near common neighborhood graph.

**Proof :** Suppose that  $S_n$  is  $i$ -balanced and  $G$  is a  $\mathcal{N}\mathcal{C}\mathcal{N}(G)$ . Then there exists a graph  $H$  such that  $\mathcal{N}\mathcal{C}\mathcal{N}(H) \cong G$ . Since  $S_n$  is  $i$ -balanced, by Theorem 1.1, there exists an  $n$ -marking  $\mu$  of  $G$  such that each edge  $uv$  in  $S_n$  satisfies  $\sigma(uv) = \mu(u)\mu(v)$ . Now consider the  $n$ -sigraph  $S'_n = (H, \sigma')$ , where for any edge  $e$  in  $H$ ,  $\sigma'(e)$  is the  $n$ -marking of the corresponding vertex in  $G$ . Then clearly,  $\mathcal{N}\mathcal{C}\mathcal{N}(S'_n) \cong S_n$ . Hence  $S_n$  is a near common neighborhood  $n$ -sigraph.

Conversely, suppose that  $S_n = (G, \sigma)$  is a near common neighborhood  $n$ -sigraph. Then there exists an  $n$ -sigraph  $S'_n = (H, \sigma')$  such that  $\mathcal{N}\mathcal{C}\mathcal{N}(S'_n) \cong S_n$ . Hence  $G$  is the  $\mathcal{N}\mathcal{C}\mathcal{N}(G)$  of  $H$  and by Theorem 2.1,  $S_n$  is  $i$ -balanced.  $\square$

In [3], the proved that, for any graph  $G$ , the near common neighborhood graph  $\mathcal{N}\mathcal{C}\mathcal{N}(G)$  and common neighborhood of  $\overline{G}$  are isomorphic. In view of this, we have the following:

**Theorem 3.4 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ ,  $\mathcal{N}\mathcal{C}\mathcal{N}(S_n)$  and  $\mathcal{C}\mathcal{N}(\overline{S_n})$  are switching equivalent.

In [3], the authors proved the following result:

**Theorem 3.5 :** For any graph  $G = (V, E)$ ,  $\mathcal{N}\mathcal{C}\mathcal{N}(G)$  and  $\mathcal{C}\mathcal{N}(G)$  are isomorphic if and only if  $G \cong \overline{G}$ .

In view of the above the result, we have the following result:

**Theorem 3.6 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ ,  $\mathcal{C}\mathcal{N}(S_n)$  and  $\mathcal{N}\mathcal{C}\mathcal{N}(S_n)$  are switching equivalent if and only if  $G \cong \overline{G}$ .

**Proof :** Suppose that  $\mathcal{C}\mathcal{N}(S_n) \sim \mathcal{N}\mathcal{C}\mathcal{N}(S_n)$ . Then clearly,  $\mathcal{C}\mathcal{N}(G) \sim \mathcal{N}\mathcal{C}\mathcal{N}(G)$ . Hence,  $G$  is a self-complementary.



Conversely, suppose that  $S_n$  is an  $n$ -sigraph whose underlying graph  $G$  is self-complementary. Then,  $\mathcal{CN}(G) \cong \mathcal{NCCN}(G)$ . Since for any  $n$ -sigraph  $S_n$ , both  $\mathcal{CN}(S_n)$  and  $\mathcal{NCCN}(S_n)$  are  $i$ -balanced, the result follows by Theorem 1.2.  $\square$

In [3], the authors remarked that:  $G \cong \mathcal{NCCN}(G)$  if and only if  $G$  is the complement of strongly regular graph which is bipartite. In view of this, we have the following result:

**Theorem 3.7 :** For any  $n$ -sigraph  $S_n = (G, \sigma)$ ,  $S_n$  and  $\mathcal{NCCN}(S_n)$  are switching equivalent if and only if  $S_n$  is  $i$ -balanced and  $G$  is the complement of strongly regular graph which is bipartite.

**Proof :** Suppose  $\mathcal{NCCN}(S_n) \sim S_n$ . This implies,  $\mathcal{NCCN}(G) \cong G$  and hence  $G$  is the complement of strongly regular graph which is bipartite. Now, if  $S_n$  is any  $n$ -sigraph with underlying graph  $G$  is the complement of strongly regular graph which is bipartite. Then  $\mathcal{NCCN}(S_n)$  is  $i$ -balanced and hence if  $S_n$  is  $i$ -unbalanced and its  $\mathcal{NCCN}(S_n)$  being  $i$ -balanced can not be switching equivalent to  $S_n$  in accordance with Theorem 1.2. Therefore,  $S_n$  must be  $i$ -balanced.

Conversely, suppose that  $S_n$  is  $i$ -balanced  $n$ -sigraph with the underlying graph  $G$  is the complement of strongly regular graph which is bipartite. Then,  $\mathcal{NCCN}(G) \cong G$ . Since  $\mathcal{NCCN}(S_n)$  is  $i$ -balanced, the result follows from Theorem 1.2 again.  $\square$

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